

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics - III

Subject Code : 4TE03EMT2

Branch: B. Tech (All)

Semester : 3

Date : 27/11/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) One of the Dirichlet's condition is function $f(x)$ should be
(A) single valued (B) multi valued (C) real valued (D) None of these
- b) If $f(x) = x^2$ is represented by Fourier series in $(-\pi, \pi)$ then b_n equal to
(A) $\pi^2/3$ (B) 0 (C) $2\pi^2/3$ (D) $\pi^2/6$
- c) In the Fourier series expansion of $f(x) = x^3$ in $(-1, 1)$
(A) only sine terms are present (B) both sine and cosine terms are present
(C) only cosine terms are present (D) constant term is present
- d) Laplace transform of e^{2t+3} is
(A) $\frac{e^3}{s-2}$ ($s > 2$) (B) $\frac{e^2}{s-3}$ ($s > 3$) (C) $\frac{1}{s-\log 2}$ (D) $\frac{1}{s-2}$
- e) Laplace transform of $\frac{\sin t}{t}$ is
(A) $\cot^{-1} \frac{1}{s}$ (B) $\tan^{-1} s$ (C) $\tan^{-1} \frac{1}{s}$ (D) $\sin^{-1} s$
- f) Inverse Laplace transform of $\frac{12}{s^2-9}$ is
(A) $3\sinh 4t$ (B) $4\sinh 3t$ (C) $4\cosh 3t$ (D) $3\cosh 4t$
- g) $\frac{1}{D-a} X$, (Where $X = k$ is constant) equal to
(A) $-\frac{k}{a}$ (B) $\frac{k}{a}$ (C) ka (D) $-ka$
- h) The C. F. of the differential equation $(D^2 - 3D + 2)y = e^{2x}$ is
(A) $c_1e^x + c_2e^{2x}$ (B) $c_1e^{-x} + c_2e^{-2x}$ (C) $c_1e^{-x} + c_2e^{2x}$ (D) $c_1e^x + c_2e^{-2x}$
- i) The P. I of $(D^2 + 6D + 5)y = 4e^{-x}$ is
(A) $4xe^{-x}$ (B) $4xe^x$ (C) xe^x (D) xe^{-x}



- j) Eliminating arbitrary function from $z = f(x^2 + y^2)$, the partial differential equation formed is
 (A) $xq = yp$ (B) $xp = yq$ (C) $z = pq$ (D) None of these
- k) The general solution of the equation $xp + yq = z$ is
 (A) $F\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ (B) $F(xy, x + y) = 0$ (C) $F\left(\frac{y}{x}, \frac{z}{y}\right) = 0$
 (D) None of these
- l) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is
 (A) $z = f_1(y + x) + f_1(y - x)$ (B) $z = f_1(y + x) + f_2(y - x)$
 (C) $z = f_2(y + x) + f_2(y - x)$ (D) $z = f(x^2 - y^2)$
- m) The order of convergence in Bisection method is
 (A) linear (B) quadratic (C) zero (D) None of these
- n) Iterative formula for finding the square root of N by Newton-Raphson method is
 (A) $x_{i+1} = \frac{1}{2}\left(x_i - \frac{N}{x_i}\right)$ ($i = 0, 1, 2, \dots$) (B) $x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$ ($i = 0, 1, 2, \dots$)
 (C) $x_{i+1} = x_i(2 - Nx_i)$ ($i = 0, 1, 2, \dots$) (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a) Using Newton-Raphson method, find the root of $f(x) = \sin x + \cos x$ correct to three decimal places. (5)
- b) One real root of the equation $x^3 - 4x - 9 = 0$ lies between 2.625 and 2.75. Find the root using Bisection method. (5)
- c) Evaluate: $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$ (4)

Q-3 Attempt all questions (14)

- a) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \leq x \leq \pi$. Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$. (5)
- b) Obtain Fourier series for the function $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$ (5)
- c) One real root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2. Find the root correct to three significant digits using Secant method. (4)

Q-4 Attempt all questions (14)

- a) Using Laplace transform method solve: (5)
 $y'' + 3y' + 2y = e^t, \quad y(0) = 1, \quad y'(0) = 0$
- b) Evaluate: $L^{-1}\left(\frac{s}{s^4 + s^2 + 1}\right)$ (5)



c) Solve: $pz - qz = z^2 + (x + y)^2$ (4)

Q-5 Attempt all questions (14)

a) Using convolution theorem evaluate: $L^{-1}\left(\frac{s}{(s^2 + 4)^2}\right)$ (5)

b) Solve: $\frac{d^3 y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}$ (5)

c) Solve: $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 2y$ (4)

Q-6 Attempt all questions (14)

a) Solve: $D^2(D^2 + 4)y = 48x^2$ (5)

b) Obtain a half – range sine series to represent $f(x) = lx - x^2$ in the range $(0, l)$. (5)

c) Evaluate: $L(t e^{2t} \cos 3t)$ (4)

Q-7 Attempt all questions (14)

a) Using the method of variation of parameters, Solve: $y'' + a^2 y = \sec ax$ (5)

b) Solve: $(x+1)^2 \frac{d^2 y}{dx^2} + (x+1) \frac{dy}{dx} + y = 2 \cos [\log(1+x)]$ (5)

c) Solve: $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$, $\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$. (4)

Q-8 Attempt all questions (14)

a) Determine the Fourier series up to and including the second harmonic to represent the periodic function $y = f(x)$ defined by the table of values given below. $f(x) = f(x + 2\pi)$ (7)

x°	0	30	60	90	120	150	180	210	240	270	300	330
$f(x)$	0.5	0.8	1.4	2.0	1.9	1.4	1.2	1.4	1.1	0.5	0.3	0.4

b) Using the method of separation of variables, (7)

solve $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, given $u(x, 0) = 6e^{-3x}$

